Ladder crystal filter design

by J. A. HARDCASTLE, G3JIR

Editorial note

This is a significant and important article, including original concepts and ideas of the author as well as a unique review of past work. Although it may appear a shade restrained and mathematical at first glance, it is a practical article that tells an amateur just how to build a high-performance hf filter at a fraction of the cost of a commercial unit—yet with virtually no constructional problems, even for newcomers who read the article carefully.

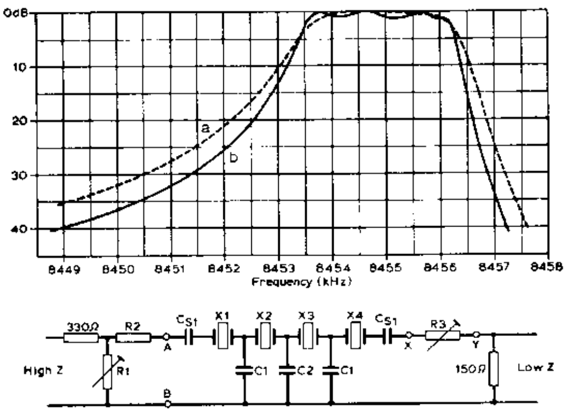
Introduction

In the four previous articles on ladder crystal filters [1] experimental results were presented without any accompanying theoretical analysis, since the practical difficulties of measuring crystal parameters accurately would have made this information valueless to most radio amateurs.

However, a simple measuring procedure has now been devised which, in conjunction with a set of capacitor coefficients, allows the construction of filters of pre-determined bandwidth. Sets of design coefficients for filters using up to eight crystals are given and accompanied by a description of their derivation.

Frequency response

Fig 1 shows two frequency response curves, the first is known as the maximally flat or Butterworth response, and the second as the equi-ripple or Chebyshev response. Ideally the number of positive peaks in this latter response should equal the number of crystals, and be of equal amplitude over the whole passband. However, in practice, fewer peaks than expected are usually found, due to some having merged with each other, and the ripple amplitude usually increases towards the band edges due to the crystals and capacitors having a finite and unequal Q; these effects being particularly marked in higher order filters.



	R1 + R2	R2	R3	C1	C2	C_{S1}
a	1625Ω	1kΩ	1625Ω	18p	27p	27p
b	706Ω	180Ω	706Ω	18p	22p	22p

Fig 1. Frequency responses of two typical 4-crystal filters, (a) Butterworth, (b) Chebyshev. The Butterworth filter bandwidth is actually 2,369Hz and the whole response has been scaled-up to allow direct comparison with the 2,762Hz bandwidth Chebyshev filter

In applications where some passband ripple is acceptable, the Chebyshev response is preferred because it has a steeper rate of cut-off and requires a lower impedance circuit than an equivalent Butterworth filter. This latter factor can be a decided advantage in circumstances which would otherwise require impracticably small capacitors.

Filter design coefficients

It has been shown previously [2,3], for Butterworth filters, that much of the labour can be taken out of filter design if each capacitor is assigned a coefficient, determining its relationship with its neighbours; and hence the filter frequency response.

Fig 2 gives design coefficients for 3rd, 4th, 6th and 8th-order Chebyshev filters which have been calculated from formulas published by P. Amstutz [4]. Actual capacitor values are derived from these coefficients by applying the formula

$$C = \frac{k \times 10^6}{2\pi f R} \quad (1)$$

where k = capacitor coefficient
f = filter centre frequency (MHz)
R = circuit impedance (ohms)
C = capacitance (pF)

In hf ladder networks it is advantageous to use shunt capacitors rather than series capacitors because this allows stray capacitance to be absorbed and allowed for in the physical components. Fig 2 shows these Type 2 filters where the input and output series capacitors have been transformed into their shunt equivalents. Unfortunately this requires an increase in circuit impedance which may not always be convenient, in which case the Type 1 filter must be used.

Filter bandwidth

One of the most important parts of a filter specification is its bandwidth, and Dishal's procedure [5] allows this to be determined from a knowledge of the crystal's equivalent series inductance which, as was said previously, is a difficult parameter to measure, and the subsequent calculations are lengthy.

Previously, a disadvantage of the simplified capacitor coefficients method has been the need to make several trial filters before the required bandwidth could be attained; however, it has now been found that these initial trials can be simplified by making systematic measurements on a 2nd-order filter. These results are then applied to whichever higher order design is required.

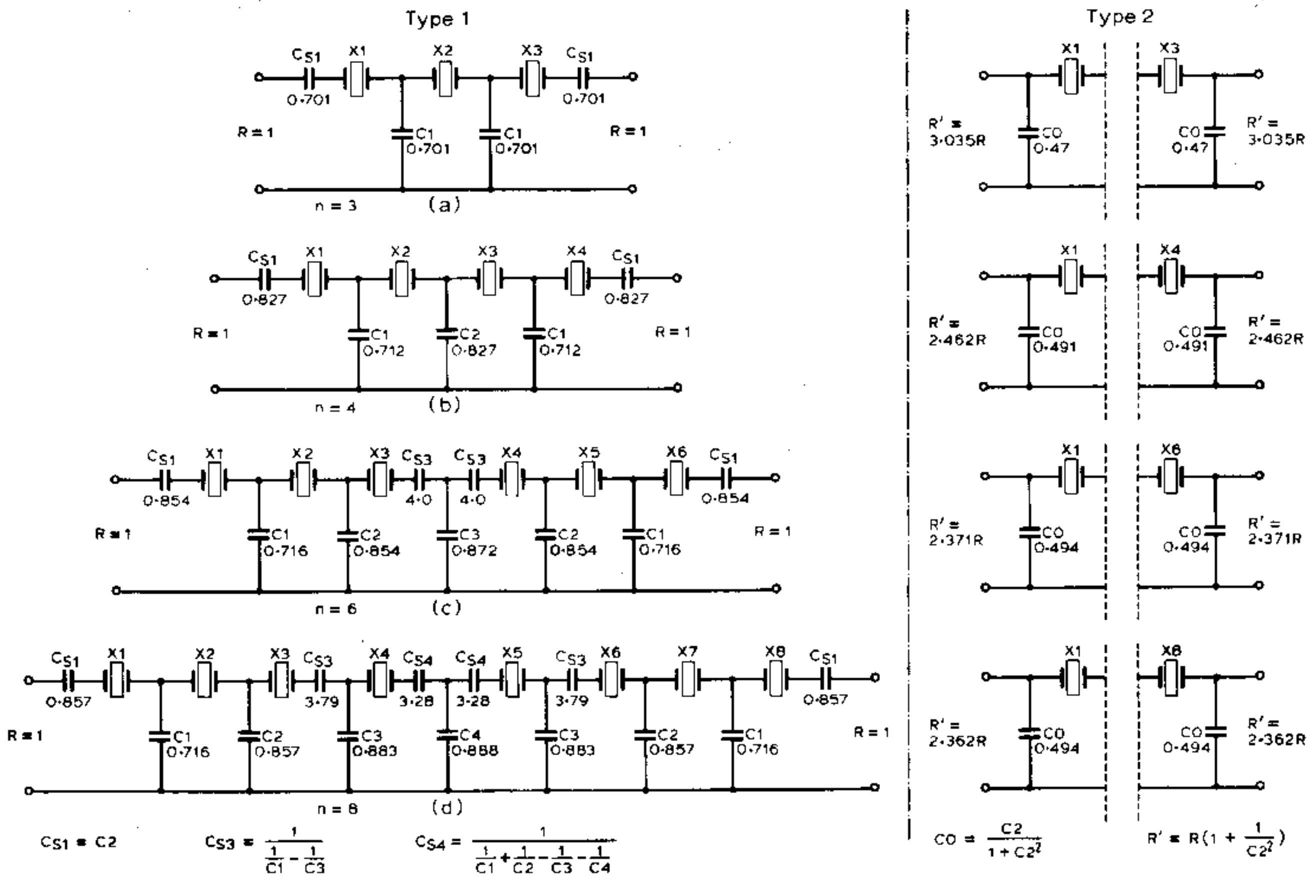


Fig 2. Design coefficients for Chebyshev filters

Initial tests

The test filter is connected as shown in Fig 3, and its frequency response and bandwidth measured using the filter test set described previously [1]. Choice of an initial value for capacitor C is arbitrary, but a value of 33pF would be suitable for many crystals. The test impedance may then be calculated by transposing equation (1).

$$R = \frac{k \times 10^6}{2\pi f C} \qquad (2)$$

$$= \frac{0 \cdot 613 \times 10^6}{2\pi \times 8 \cdot 454 \times 33} = 349 \cdot 7\Omega$$

The test set input and output impedances are now set to this value. To set the input impedance, measure across A–B while adjusting R3; to set the output impedance, measure across points X and Y and adjust R4. The different input and output circuits are necessitated by the test set's particular circuit arrangements.

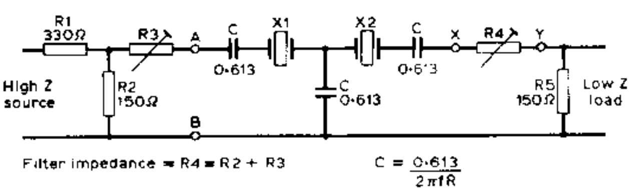


Fig 3. Preliminary tests to determine filter bandwidth use this circuit in conjunction with the filter test set (see text)

A typical frequency response curve obtained by this method, Fig 4, is seen to have a dip of very nearly the theoretical 1dB in the centre. Ideally the peaks on either side should be equal, but rarely are, due to minor differences between the two crystals. However, the most important parameter, the bandwidth, is well defined and easily measured because the response is falling rapidly at the 3dB-down points.

Filter bandwidth has been found to be inversely proportional to the square root of the coupling capacitance, and once an

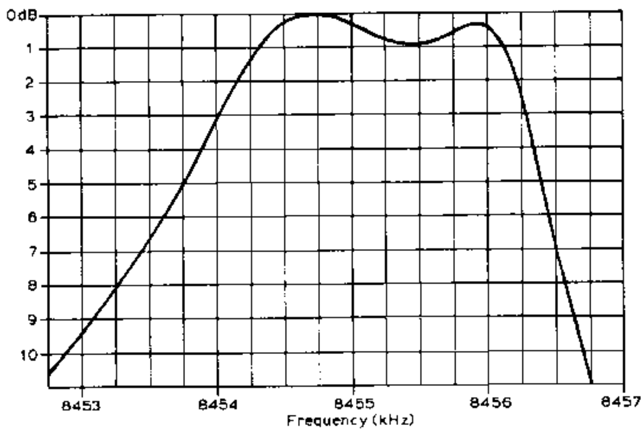


Fig 4. A typical test filter response. The dip in the centre of the passband is 0·9dB and the bandwidth 2,287Hz. Ideally both peaks would be equal and the central dip 1dB

initial measurement has been made a very close approximation to the correct capacitance may be calculated from (3).

$$C_2 = C_1 \times \left(\frac{BW_1}{BW_2}\right)^2 \qquad (3)$$

where C_1 and BW_1 are the capacitance and bandwidth found in the first measurement, and BW_2 is the design objective.

Equation (2) is used again to determine the new value of impedance to be used with C_2. If these components prove to give the desired bandwidth, this impedance is used to calculate the required higher order filter from the coefficients given in Fig 2.

Design example

A 6th-order Chebyshev filter will now be designed to illustrate the application of the procedures described so far.

From Table 1 the components giving the bandwidth nearest to 2,400Hz are selected and C_2 calculated.

$$C_2 = C_1 \times \left(\frac{BW_1}{BW_2}\right)^2 = 18 \times \left(\frac{2287}{2400}\right)^2 = 16{\cdot}34\text{pF}$$

The new circuit impedance is then calculated

$$R = \frac{k \times 10^6}{2\pi fC} = \frac{0{\cdot}613 \times 10^6}{2\pi \times 8{\cdot}454 \times 16{\cdot}34} = 706\Omega$$

Using this value for R, the filter capacitors can be calculated from the coefficients given in Fig 2

$$C_1 = \frac{k_1 \times 10^6}{2\pi fR} = \frac{0{\cdot}7159 \times 10^6}{2\pi \times 8{\cdot}454 \times 706} = 19{\cdot}1\text{pF}$$

Similarly $C_2 = 22{\cdot}7$pF, $C_3 = 23{\cdot}2$pF and $C_{s3} = 106{\cdot}7$pF. This filter was constructed using miniature wire-ended crystals and preferred value capacitors and is shown, with its frequency response, in Fig 5. Note that the bandwidth at −3dB is 2,580Hz, which is considered to be sufficiently close to the design objective for amateur purposes.

Components

Choice of crystals is largely limited to whatever is available cheaply, but it has been found that the miniature wire-ended crystals require a circuit impedance which is higher than for HC6U types. Although very satisfactory filters have been made using these miniature types, the high impedance circuit is more vulnerable to stray capacitance and some individual capacitance trimming may be necessary to achieve the best performance. Therefore, when available, HC6U crystals are preferred.

Table 1. Test measurements made on a pair of crystals using various capacitors (C) and circuit impedances (R). In each case R was calculated from equation (2)

R (Ω)	C (pF)	f_1 (kHz)	f_2 (kHz)	Bandwidth (Hz)	Ripple (dB)
769	15	8,454·198	8,456·742	2,544	1·3
641	18	8,454·001	8,456·288	2,287	0·9
525	22	8,453·837	8,455·912	2,075	0·8
427	27	8,453·742	8,455·611	1,869	0·9
350	33	8,453·660	8,455·333	1,673	0·9

Capacitors may be polystyrene or silvered mica types. Where very small capacitances are called for, a trimmer may be adjusted to the required value or a short piece of miniature coaxial cable may be cut to the required length [6].

The evolution of a ladder crystal filter

Fig 6 shows successive stages in the evolution of a ladder crystal filter. The initial low-pass prototype, Fig 6(a), is converted into a bandpass filter, Fig 6(b), by adding an inductor to parallel-resonate each shunt capacitor to the centre frequency. Similarly each series inductor is series resonated by adding a series capacitor.

The next stage of the process uses an impedance inverter, Fig 6(c), which converts a shunt, parallel-resonant circuit into a series, series-resonant circuit. Although the impedance inverter uses a negative capacitor in its series arms, this is later absorbed by other, more positive capacitors, so there are no physically unrealizable capacitors in the final design.

This procedure was described by S. B. Cohn [7] for use in the design of coupled resonator filters and was applied to crystal filters by P. Amstutz [4] by assuming that, for narrow-band filters, the series resonant circuit within the dotted line in Fig 6(e) is approximated with sufficient accuracy by a piezoelectric crystal.

Chebyshev filter coefficient calculation

The Amstutz calculations may be illustrated by the following calculation of the coefficients for a 3rd-order filter.

Let the ripple amplitude be $a = \text{dB}$

and $e = 2{\cdot}718$

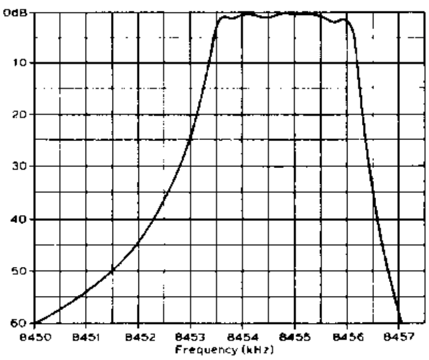
and n = number of crystals

$$\text{calculate } m = \frac{a}{8{\cdot}686}$$

$$s = e^m$$

$$t = \frac{1}{n} \text{ arctanh } \frac{1}{s}$$

If a = 1dB and n = 3 then t = 0·476.



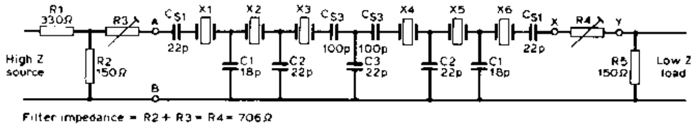


Fig 5. 6-pole Chebyshev filter. The bandwidth at −3dB is 2,581Hz and at −60dB is 7,002Hz. Note that there are only five peaks in the passband instead of the theoretical six

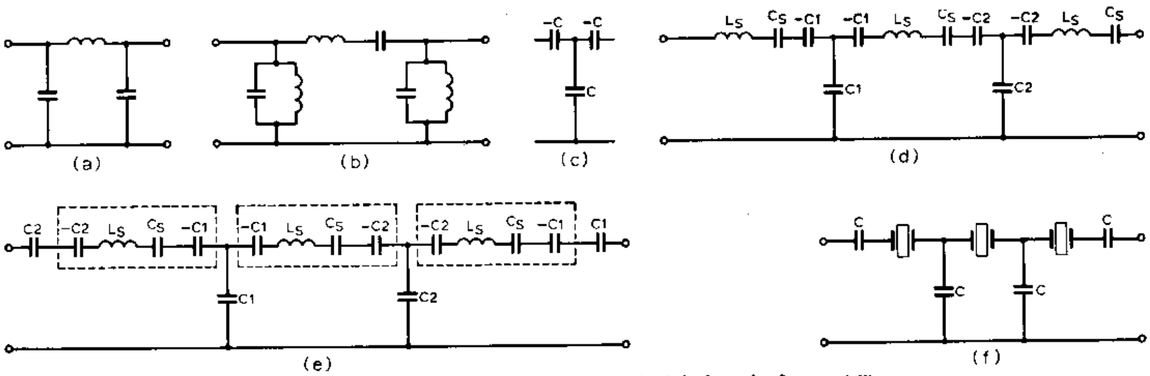


Fig 6. Stages in the evolution of the theoretical design of a 3-crystal filter

The circuit impedance coefficient is now calculated from

$$R = \frac{\sinh t}{\sin\left(\frac{180}{2n}\right)} = 0{\cdot}988 \qquad (4)$$

and the coupling capacitor coefficients are calculated from

$$C_b = \sqrt{\frac{\cos\left(\frac{180}{n}\right) - \cos\left(\frac{360\,b}{n}\right)}{\cosh 2t - \cos\left(\frac{360\,b}{n}\right)}} \qquad (5)$$

for b = 1, 2, ——— (n − 1)

Hence $C_1 = 0{\cdot}7092$ and $C_2 = 0{\cdot}7092$.

The filter now appears as in Fig 7(a). This is normalized for a circuit impedance of 1Ω by dividing the impedance by 0·988 and multiplying all the capacitors by the same amount, with the result shown in Fig 7(b).

As mentioned earlier, the series input capacitors may be replaced by shunt capacitors, C_0 in Fig 7 (c), and these are derived by the simple calculation shown in this diagram. In this example the impedance is increased by 3·035 by the circuit rearrangement. Colin [8] and F6BQP [3] took this calculation one stage further by again normalizing for an impedance of 1Ω, but this has not been done here in order to preserve the simple relationship between the Type 1 filter and the test filter.

Butterworth filters

It is not necessary to give full details of the derivation of Butterworth filter coefficients because they follow a similar procedure to the previous paragraph. However, for completeness, the Amstutz formulas are given below so that anyone who wishes may confirm for themselves the coefficients published previously.

$$C_b = \sqrt{\frac{\cos\left(\frac{180}{n}\right) - \cos\left(\frac{360\,b}{n}\right)}{2}} \qquad (6)$$

for b = 1, 2 ——— (n − 1)

$$R = \frac{1}{\sin\left(\frac{90}{n}\right)} \qquad (7)$$

Conclusion

Design coefficients have been presented for a range of filters which should satisfy most amateur requirements. They have been tested by constructing filters using 2, 3, 4, 6 and 8 crystals, and the results of these measurements confirm that they behave in a virtually identical manner to filters made from Dishal's design. However, it must be noted that this simplified design method is limited to filters having relatively symmetrical frequency characteristics, and single sideband filters must be designed using Dishal's more comprehensive design.

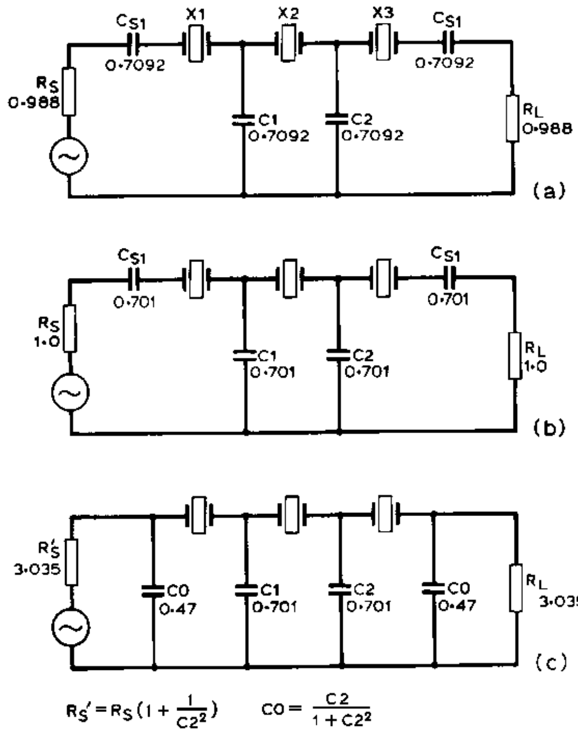


Fig 7. Three stages in the calculation of a set of design coefficients. (a) Coefficients obtained from equations (4) and (5). (b) Coefficients for the Type 1 filter normalized for 1Ω impedance. (c) Coefficients for the Type 2 filter

Now that most of the experimental element has been removed from this simple procedure it is hoped that more amateurs, particularly beginners, will be encouraged to construct their own filters, especially when inflation has placed commercial products almost beyond reach.

Acknowledgements

It is wished to acknowledge the many sources of information listed in the references, all have made their own contribution to an understanding of ladder crystal filters, without which the simplified method of predicting filter bandwidth could not have been developed.

It is also wished to acknowledge the expert assistance of Miss A. E. Howarth for carrying out literature searches and making translations.

References

[1] "Some experiments with high frequency ladder crystal filters", J. A. Hardcastle, G3JIR. *Radio Communication* December 1976, January, February and September 1977.

[2] "Formulas for the calculation of narrow bandpass filters with identical piezoelectric crystals and maximally flat attenuation behaviour", J. E. Colin (in French). *Cables and Transmission*, Vol 22, April 1968, pp132-5.

[3] "Crystal ladder filters", J. Pochet, F6BQP. In "Technical Topics" *Radio Communication* September 1976; *Wireless World* July 1977.

[4] "Narrow band filters", P. Amstutz (in French). *Cables and Transmission*, Vol 21, April 1967, pp88-97.

[5] "Modern network theory design of single sideband crystal ladder filters", M. Dishal. *Proc IEEE*, Vol 53, September 1965, pp1205-16.

[6] "Technical Topics", J. P. Hawker. *Radio Communication* September 1977, p693.

[7] "Direct coupled resonator filters", S. B. Cohn. *Proc IRE* February 1957.

[8] "Narrow bandpass filters using identical crystals designed by the image parameter method", J. E. Colin (in French). *Cables and Transmission*, Vol 21, April 1967, pp132-5.

[9] "Modern filter theory and design", editors G. C. Temes and S. K. Mitra, Chapter 4 "Crystal and ceramic filters", G. Szentirmai.

[10] Private communication, J. Haine, ex-G8CEG, 22 April 1977.

□